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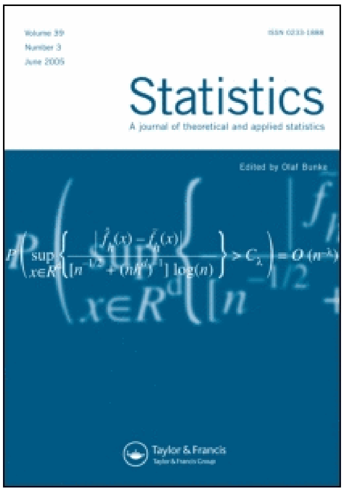

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A Class of Large Sample Tests for Bivariate Exponentiality Versus BIFRA and BNBU-(t_0, t_0) Alternatives

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Summary. Two related problems are considered here. First, a class of tests is proposed for testing bivariate exponentiality (BVE) against the class of bivariate increasing failure rate average (IFRA) distributions. Secondly, we propose a test of BVE versus bivariate new better than used of age (t_0, t_0) (BNBU-(t_0, t_0)), a new class of bivariate distributions introduced here. The PITMAN asymptotic relative efficiencies of these tests with respect to BASU and EBRAHIMI's (1984) bivariate new better than used (BNBU) test, are compared.

Key words: Bivariate exponential distribution, increasing failure rate average, new better than used, new better than used of age (t_0, t_0), asymptotic relative efficiency.

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1. Introduction

In reliability theory, life distributions have been classified based upon the notion of aging, or upon the wear-out characteristics of the item whose life length is of interest. Chief among these classes are: (1) Increasing failure rate (IFR), (2) Increasing failure rate average (IFRA), (3) New better than used (NBU), (4) New better than used in expectation (NBUE) and so on (cf. eg. BARLOW and PROSCHAN (1975)). Multivariate versions of IFR, IFRA, NBU, NBUE etc. have also been defined and their properties have been studied. For example, see, BUCHANAN and SINGPURWALLA (1977) and BARLOW and PROSCHAN (1977). The problem of testing for bivariate new better than used distribution has been discussed by BASU and EBRAHIMI (1984). In 1977, BARLOW and PROSCHAN analyzed bivariate failure data on Caterpillar tractors using graphical techniques and concluded that the life distribution is probably a bivariate IFRA. Motivated by this example, in this paper we consider testing for bivariate exponentiality versus a class of distributions with bivariate IFRA property. Let X and Y be the life lengths of two devices with the joint distribution function $F(x, y)$ with $\bar{F}(0, 0)=1$, where $\bar{F}(x, y)=\Pr(X>x, Y>y)$ is the joint survival function.

Definition 1.1. A life distribution F is Bivariate IFRA (BIFRA) if and only if, for all $x \geqq 0$, $y \geqq 0$, $0<\alpha<1$,

$$\bar{F}(\alpha x, \alpha y) \geqq \{\bar{F}(x, y)\}^{\alpha}. \tag{1.1}$$

Definition 1.2. A life distribution F is said to be Bivariate NBU (BNBU) if and only if, for all $x, y, t \geqq 0$

$$\bar{F}(x+t, y+t) \leqq \bar{F}(x, y)\, \bar{F}(t, t)\,. \tag{1.2}$$

The equality in (1.1) and (1.2) holds if and only if F is bivariate exponential distribution (BVE) introduced by Marshall and Olkin (1967) (see equation (1.4)). The definition (1.2) appears in Buchanan and Singpurwalla (1977). The dual classes, namely, the bivariate decreasing failure rate average distributions (BDFRA) and the bivariate new worse than used distributions (BNWU) are defined by reversing the inequalities in (1.1) and (1.2) respectively.

We now introduce a new class of bivariate life distribution, namely bivariate NBU (t_0, t_0) (BNBU-(t_0, t_0)), where the joint survival probability at age (0, 0) is greater than or equal to the joint conditional survival probability at specified age (t_0, t_0), $t_0 > 0$.

Definition 1.3. Let $t_0 > 0$ be fixed. A bivariate life distribution F is BNBU-(t_0, t_0) if, for all $x, y \geqq 0$,

$$\bar{F}(x+t_0, y+t_0) \leqq \bar{F}(x, y)\, \bar{F}(t_0, t_0)\,, \tag{1.3}$$

where $\bar{F}$ denotes the joint survival function. The dual class, bivariate new worse than used of age (t_0, t_0) (BNWU-(t_0, t_0)) is defined by reversing the inequality in (1.3). Let

$$\xi_0 = \{F : \bar{F}(x+t_0, y+t_0) = \bar{F}(x, y)\, \bar{F}(t_0, t_0), \quad \text{for all} \quad x, y \geqq 0\}$$

and

$$\xi_A = \{F : \bar{F}(x+t_0, y+t_0) \leqq \bar{F}(x, y)\, \bar{F}(t_0, t_0), \quad \text{for all} \quad x, y \geqq 0 \text{ and strict inequality holds for some } x, y \geqq 0\}\,.$$

Then ξ_0 is the class of boundary members of BNBU-(t_0, t_0) and BNWU-(t_0, t_0) classes and includes, in the main, the BVE distribution (1.4). This class BNBU-(t_0, t_0) is a bivariate extension of the (univariate) NBU-t_0 introduced in Hollander, Park and Proschan (1986) to which we refer the reader for some examples. Note that the BNBU-(t_0, t_0) class contains the class of BNBU distributions.

In Section 2, a class of statistics is proposed to test

$$H_0 : \bar{F}(x, y) = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_{12} \max(x, y)\}, \quad x, y \geqq 0, \quad \lambda_1, \lambda_2, \lambda_{12} > 0 \tag{1.4}$$

versus

$$H_1 : F \text{ is BIFRA} \quad \text{(and not BVE).} \tag{1.5}$$

The asymptotic behavior and consistency of test statistics are also discussed in this section. In section 3, a test for testing

$$H_0' : F \text{ is in } \xi_0$$

versus

$$H_A : F \text{ is in } \xi_A$$

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is introduced. Finally, in Section 4, the large sample properties of the tests discussed in Section 2 and 3 are studied using PITMAN's asymptotic relative efficiency.

2. A Class of BIFRA tests

Let $(X_1, Y_1), \ldots, (X_n, Y_n)$ be a random sample of size n from the distribution F. To test H_0 in (1.4) versus the alternative H_1 in (1.5) consider the parameter

$$\mu_\alpha(F) = \int_0^\infty \int_0^\infty \bar{F}(\alpha x, \alpha y)\, dF(x, y) . \tag{2.1}$$

When $F \in H_0$, $\mu_\alpha(F) = (2 + \lambda_{12}(\alpha+1))/\lambda(\alpha+1)^2$, and, for all $F \in H_1$, $\mu_\alpha(F) \geqq (2+\lambda_{12}(\alpha+1))/\lambda(\alpha+1)^2$, where $\lambda = \lambda_1 + \lambda_2 + \lambda_{12}$. Hence, viewing $\left(\mu_\alpha(F) - \frac{2+\lambda_{12}(\alpha+1)}{\lambda(\alpha+1)^2}\right)$ as a measure of deviation of F from H_0 towards H_1, the usual nonparametric approach of replacing F in $\mu_\alpha(F)$ by F_n, where

$$F_n(x, y) = \frac{1}{n} \sum_{i=1}^{n} I[X_i \leqq x,\ Y_i \leqq y] \tag{2.2}$$

is the bivariate empirical distribution function, suggests the rejection of H_0 in favor of H_1 for large values of $\mu_\alpha(F_n)$. Observe that (cf. (2.1))

$$\mu_\alpha(F_n) = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} I[X_i > \alpha X_j]\, I[Y_i > \alpha Y_j] .$$

Here and in (2.2), $I[A]$ denotes the indicator function of the set A. Note that $\mu_\alpha(F_n)$ is asymptotically equivalent to the U-statistic:

$$J_n^\alpha = \frac{1}{n(n-1)} \sum_{1 \leqq i \neq j \leqq n} h_\alpha\{(X_i, Y_i)(X_j, Y_j)\} ,$$

where

$$h_\alpha\{(X_i, Y_i)(X_j, Y_j)\} = \begin{cases} 1 & \text{if } X_i > \alpha X_j \text{ and } Y_i > \alpha Y_j \\ 0 & \text{o.w.} \end{cases}$$

and the test rejects H_0 in favor of H_1 for large values of J_n^α. It follows from the results of HOEFFDING (1948) that the asymptotic distribution of $\sqrt{n}\,(J_n^\alpha - \mu_\alpha(F))$ is normal with mean 0 and variance $4\zeta_1$, where

$$\zeta_1 = \mathsf{E}[\Psi_1^2(X_1, Y_1)] - [\mu_\alpha(F)]^2 , \tag{2.3}$$

and

$$\Psi_1(x_1, y_1) = \mathsf{E}\{h_\alpha^*((x_1, y_1), (X_2, Y_2))\}, \quad \text{provided} \quad \zeta_1 > 0 .$$

Here h_α^* is the symmetric version of h_α. Under H_0, $\mu_\alpha(F) = \frac{2+\lambda_{12}(\alpha+1)}{\lambda(\alpha+1)^2}$, and ζ_1 is a function of λ_1, λ_2 and λ_{12}.

Since the variance of $\sqrt{n}\,J_n^\alpha$ under H_0 is very complicated, jackknifing could be used to estimate this quantity.

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Let $J^{\alpha}_{n,i} = J^{\alpha}_{n-1}((X_1, Y_1), \ldots, (X_{i-1}, Y_{i-1}), (X_{i+1}, Y_{i+1}), \ldots, (X_n, Y_n))$ and $J^{*\alpha}_n = \frac{1}{n}\sum_{i=1}^{n} J^{\alpha}_{n,i}$. The estimate of $V(\sqrt{n}J^{\alpha}_n)$ is

$$\hat{V}(\sqrt{n}J^{\alpha}_n) = \frac{n}{n-1}\sum_{i=1}^{n}(J^{\alpha}_{n,i} - J^{*\alpha}_n)^2$$

and it follows from the results of SEN (1977) that $\frac{\sqrt{n}\,(J^{\alpha}_n - \mu_\alpha(F))}{[\hat{V}(\sqrt{n}J^{\alpha}_n)]^{1/2}}$ is asymptotically standard normal. We now illustrate this test by applying it to the failure data on Caterpillar tractors given in BARLOW and PROSCHAN (1977). Let α' denote the level of significance. In particular for $\alpha = 0.9$, we obtain $J^{0.9}_{15} = 0.4333$, $\hat{V}(\sqrt{15}J^{0.9}_{15}) = 0.1674$ and $(15)^{1/2}(J^{0.9}_{15})\{\hat{V}(\sqrt{15}J^{0.9}_{15})\}^{-\frac{1}{2}} = 4.1024$, and for $\alpha' = 0.01$, H_0 is rejected in favor of H_1.

Remark 2.1. It is clear that J^{α}_n is consistent against the BIFRA alternatives.

3. A BNBU-(t_0, t_0) test

In this section, a statistic for testing

$$H_0' : F \text{ is in } \xi_0$$

versus

$$H_A : F \text{ is in } \xi_A$$

based on a random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$ of size n, is introduced. Under H_0' a new two-component device is as good as a used two-component device, of which each component is of age t_0, whereas under H_A, a used two-component device of age (t_0, t_0) has stochastically smaller residual life than a new one.

Consider the parameter

$$\nabla(F) = \int_0^\infty \int_0^\infty [\bar{F}(x, y)\,\bar{F}(t_0, t_0) - \bar{F}\,(x+t_0, y+t_0)]\,\mathrm{d}F(x, y) \tag{3.1}$$

$$= \bar{F}(t_0, t_0)\int_0^\infty \int_0^\infty \bar{F}(x, y)\,\mathrm{d}F(x, y) - \int_0^\infty \int_0^\infty \bar{F}\,(x+t_0, y+t_0)\,\mathrm{d}F(x, y)$$

$$\stackrel{\text{def}}{=} \nabla_1(F) - \nabla_2(F)\,.$$

Note that under H_0', $\nabla(F) = 0$, and under $H_A : \nabla(F) \geqq 0$. Hence it is reasonable to reject H_0' in favor of H_A when $\nabla(F_n)$ is too large, where $\nabla(F_n)$ is obtained by replacing F in (3.1) by the bivariate empirical distribution function F_n defined in (2.2). However, it is more convenient to reject H_0' in favor of H_A for large values of the asymptotically equivalent U-statistic given by

$$U_n = \frac{1}{n\,(n-1)\,(n-2)}\sum_{i \neq j \neq k} t(X_i, t_0)\,t(Y_j, t_0)\,t(X_j, X_k)\,t(Y_j, Y_k)$$

$$- \frac{1}{n\,(n-1)}\sum_{i \neq j} t\,(X_i, X_j + t_0)\,t\,(Y_j, Y_j + t_0) \stackrel{\text{def}}{=} U_{1n} - U_{2n}\,,$$

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where

$$t(a, b) = 1 \quad \text{if} \quad a > b$$
$$= 0 \quad \text{o.w.}$$

Let $\varphi_1((X_1, Y_1), (X_2, Y_2)) = \frac{1}{2} [t(X_1, X_2 + t_0)\, t(Y_1, Y_2 + t_0) + t(X_2, X_1 + t_0)\, t(Y_2, Y_1 + t_0)]$

and

$$\varphi_2((X_1, Y_1), (X_2, Y_2), (X_3, Y_3))$$
$$= \frac{1}{6} [t(X_1, t_0)\, t(Y_1, t_0)\, t(X_2, X_3)\, t(Y_2, Y_3)$$
$$+ t(X_1, t_0)\, t(Y_1, t_0)\, t(X_3, X_2)\, t(Y_3, Y_2)$$
$$+ t(X_2, t_0)\, t(Y_2, t_0)\, t(X_1, X_3)\, t(Y_1, Y_3)$$
$$+ t(X_2, t_0)\, t(Y_2, t_0)\, t(X_3, X_1)\, t(Y_3, Y_1)$$
$$+ t(X_3, t_0)\, t(Y_3, t_0)\, t(X_1, X_2)\, t(Y_1, Y_2)$$
$$+ t(X_3, t_0)\, t(Y_3, t_0)\, t(X_2, X_1)\, t(Y_2, Y_1)]$$

be symmetric kernels of degree 2 and 3 corresponding to $\nabla_1(F)$ and $\nabla_2(F)$, respectively. Then, using the results of HOEFFDING (1948), we know that the limiting distribution of $\sqrt{n}\,(U_n - \nabla(F))$ is normal with mean 0 and variance $\sigma^2 = 4\zeta_1^{(1)} + 9\zeta_1^{(2)} - 12\zeta_1^{(12)}$, where

$$\zeta_1^{(1)} = \mathsf{E}\,\{\mathsf{E}\,(\varphi_1((X_1, Y_1), (X_2, Y_2)) \mid X_1 = x_1, Y_1 = y_1)\}^2 - \nabla_1^2(F)\,,$$
$$\zeta_1^{(2)} = \mathsf{E}\,\{\mathsf{E}\,(\varphi_2((X_1, Y_1), (X_2, Y_2), (X_3, Y_3)) \mid X_1 = x_1, Y_1 = y_1)\}^2 - \nabla_2^2(F)\,,$$
$$\zeta_1^{(12)} = \mathsf{E}\,\{[\mathsf{E}\,(\varphi_1((X_1, Y_1), (X_2, Y_2)) \mid X_1 = x_1, Y_1 = y_1) - \nabla_1(F)]$$
$$[\mathsf{E}\,(\varphi_2((X_1, Y_1), (X_2, Y_2), (X_3, Y_3)) \mid X_1 = x_1, Y_1 = y_1) - \nabla_2(F)]\}\,.$$

Note that under H_0', the mean of $\sqrt{n}\,U_n$ is 0 but the null asymptotic variance of $\sqrt{n}\,U_n$ depends on F and hence must be estimated from the data. Once again jackknifing serves the purpose.

Let

$$U_{n,l} = U_{n-1}((X_1, Y_1), (X_2, Y_2), \ldots, (X_{l-1}, Y_{l-1}), (X_{l+1}, Y_{l+1}), \ldots (X_n, Y_n)),$$
$$U_n^* = \frac{1}{n} \sum_{l=1}^{n} U_{n,l}\,,$$

and

$$\hat{\sigma}^2 = \frac{n}{n-1} \sum_{l=1}^{n} [U_{n,l} - U_n^*]^2\,.$$

It then follows from the results of SEN (1977) that $\hat{\sigma}^2$ is a consistent estimator of σ^2, and $\sqrt{n}\,U_n \hat{\sigma}^{-1}$ is asymptotically standard normal. Thus, the approximate α-level test of H_0' against H_A (refered to as BNBU-(t_0, t_0) test) rejects H_0 in favor of H_A if $\sqrt{n}\,U_n \hat{\sigma}^{-1} \geqq z_\alpha$ where z_α is the upper α percentile of a standard normal distribution. It can be shown, along the lines of HOLLANDER, PARK and PROSCHAN (1986) that BNBU-(t_0, t_0) test is consistent against all continuous alternatives.

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4. Asymptotic relative efficiency

In this section we compare the BIFRA test with (i) the BNBU test proposed by Basu and Ebrahimi (1984) and (ii) the BNBU-(t_0, t_0) test.

Let $\{F_{\vartheta_n}\}$ be a sequence of alternatives with $\vartheta_n = \vartheta_0 + \frac{a}{\sqrt{n}}$, where a is any arbitrary positive constant and F_{ϑ_0} is BVE with parameters (1, 1, 1). From the results of Basu and Ebrahimi (1984) and those in Section 2, the Pitman asymptotic relative efficiency (ARE) of the BIFRA test with respect to Basu and Ebrahimi's (1984) test is given by

$$e_F(J_n^\alpha, J_n) = \left\{\frac{\text{eff}\,(J_n^\alpha)}{\text{eff}\,(J_n)}\right\}^2 = \left\{\frac{\mu'(\vartheta_0)}{\Delta'(\vartheta_0)}\right\}^2 \frac{\sigma^2_{H_0}(J_n)}{\sigma^2_{H_0}(J_n^\alpha)}, \tag{4.1}$$

where eff (J_n^α) and eff (J_n) denote the efficacies of tests, based on J_n^α and J_n, respectively. In (4.1) $\sigma^2_{H_0}(J_n)$ and $\sigma^2_{H_0}(J_n^\alpha)$ are null asymptotic variances of $\sqrt{n}J_n$ and $\sqrt{n}J_n^\alpha$ respectively;

$$\mu(\vartheta) = \int_0^\infty \int_0^\infty \bar{F}_\vartheta(\alpha x, \alpha y)\, \mathrm{d}F_\vartheta(x, y)$$

$$\Delta(\vartheta) = \frac{1}{2} \int_0^\infty \int_0^\infty \bar{F}_\vartheta(x, y)\, \mathrm{d}F_\vartheta(x, y) - \int_0^\infty \int_0^\infty \int_0^\infty \bar{F}_\vartheta\,(x+t, y+t) \times \mathrm{d}F_\vartheta(x, y)\, \mathrm{d}F_\vartheta(t, t)$$

are the asymptotic means of J_n^α and J_n respectively, for the alternative F_ϑ and $\mu'(\vartheta_0)$ $(\Delta'(\vartheta_0))$ is the derivative of $\mu(\vartheta)$ $(\Delta(\vartheta))$ with respect to ϑ evaluated at $\vartheta = \vartheta_0$. Consider the Weibull alternative given by

$$\bar{F}_\vartheta(x, y) = \exp\{-x^\vartheta - y^\vartheta - \max(x^\vartheta, y^\vartheta)\}, \quad \vartheta \geqq 1, \quad x \geqq 0, \quad y \geqq 0\,.$$

Note that BVE(1, 1, 1) corresponds to $\vartheta = 1$. After long and tedious computations, one can demonstrate that

$$\mu'(\vartheta_0) = \frac{(\alpha+1)^{-3}}{3}\left[(\alpha+1)^2\{(C-1) + \log 3 + \log(1+\alpha)\} - (5+\alpha)\,\alpha\log\alpha\right] \tag{4.2}$$

$$\Delta'(\vartheta_0) = 0.4096\,, \tag{4.3}$$

respectively, where in (4.2) $C = 0.5772\ldots$ is the Euler constant (C.f. Ryzhik and Gradshteyn (1957)).

The null asymptotic variance of $\sqrt{n}J_n^\alpha$ using (2.3) is

$$\begin{aligned}\sigma^2_{H_0}(J_n^\alpha) = \{&1 + 3^{-1}(\alpha+2)^{-2}\,\alpha\,(9\alpha+14) + 3^{-1}(\alpha+1)^{-2}(6 - 8\alpha - 6\alpha^2)\\ &+ 2\alpha\,(3\alpha+4)^{-1}(1 + 2\alpha\,(\alpha+1)^{-1}) - 4\alpha\,(5+3\alpha)^{-1}(\alpha\,(\alpha+2)^{-1}\\ &+ \alpha\,(\alpha+1)^{-1} + 1) + (2\alpha+3)\,3^{-1}(2\alpha+1)^{-2} - 4\alpha\,(3\alpha^2+3\alpha+2)^{-1}\\ &\times(\alpha\,(\alpha^2+\alpha+1)^{-1} + (\alpha+1)^{-1} + 1) + 2\alpha\,(\alpha^2+3\alpha+1)\,3^{-1}\\ &\times(\alpha^2+\alpha+1)^{-2} - 4\,(\alpha+3)^2\,9^{-1}(\alpha+1)^{-4}\}\,,\end{aligned} \tag{4.4}$$

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and the null asymptotic variance of $\sqrt{n}J_n$, using equation (3.1) of BASU and EBRAHIMI (1984), can be shown to be

$$\sigma^2_{H_0}(J_n) = 0.2664\ , \tag{4.5}$$

approximately.

The PITMAN ARE of BASU and EBRAHIMI'S (1984) test with respect to the BNBU-(t_0, t_0) test for the Weibull alternative is given by

$$e_F(J_n, U_n) = \left\{\frac{\Delta'(\vartheta_0)}{\nabla'(\vartheta_0)}\right\}^2 \frac{\sigma^2_{H_0}(U_n)}{\sigma^2_{H_0}(J_n)}\ ,$$

where $\Delta'(\vartheta_0)$ and $\sigma^2_{H_0}(J_n)$ are given by (4.3) and (4.5) respectively;

$$\nabla'(\vartheta_0) = \frac{\mathrm{d}}{\mathrm{d}\vartheta}\left\{\int_0^\infty\int_0^\infty \bar{F}_\vartheta(x+t_0, y+t_0)\,\mathrm{d}\bar{F}_\vartheta(x, y) - \bar{F}_\vartheta(t_0, t_0)\int_0^\infty\int_0^\infty \bar{F}_\vartheta(x, y)\,\mathrm{d}F_\vartheta(x, y)\right\}$$

$$= 4^{-1}\exp(-3t_0)\,[\exp(4t_0)\,E_i(-4t_0) - 4 - \log 4t_0]$$

with

$$E_i(-x) = -\int_x^\infty \frac{e^{-t}}{t}\,\mathrm{d}t\ ,$$

and

$$\sigma^2_{H_0}(U_n) = \{0.4622 + 1.1284\exp(-3t_0) - 1.1700\exp(-5t_0) - 0.0568\exp(-6t_0)\}\exp(-3t_0)\ .$$

Table 1.
Efficacies of J_n^α, and ARE's of J_n^α tests with respect to J_n test

α	eff (J_n^α)	$e_F(J_n^\alpha, J_n)$
0.175	2.3728	8.9417
0.255	2.3768	8.9721
0.5	2.2254	7.8652
0.9	1.4145	3.1775

Table 2.
ARE of J_n test with respect to the BNBU-(t_0, t_0) test, U_n

t_0	$e_F(J_n, U_n)$
0.25	7.6727
0.50	9.6935
1.00	21.7741
1.50	67.9711
2.00	246.4543
2.50	955.0060

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Tables 1 and 2 give the ARE's of J_n^α tests with respect to J_n test for $\alpha = .175$, .255, .5 and .9, and the ARE of J_n test with respect to U_n test for $t_0 = 0.25, 0.5, 1.0, 1.5, 2.0$ and 2.5. They indicate that the ARE's $e_F(J_n^\alpha, J_n)$ and $e_F(J_n, U_n)$ are quite high as to be expected since J_n^α test is designed for a smaller class of alternatives than the J_n test and in turn J_n test is designed for smaller class of alternatives than the BNBU-(t_0, t_0) test, U_n.

It can be verified that the efficacy of J_n^α has an asymptote at $\alpha = 0$. Except for this, the efficacy of J_n^α is a maximum at $\alpha = 0.255$. Thus, one may recommend $J_n^{0.255}$ for testing BIFRA alternatives.

The details of computations in this section are not shown here but can be obtained from the authors.

References

BARLOW, R. E. and PROSCHAN, F. (1975). *Statistical Theory of Reliability and Life Testing* Holt, Rinehart and Winston, Inc., New York.

BARLOW, R. E. and PROSCHAN, F. (1977). Techniques for analyzing multivariate failure data. *Theory and Applications of Reliability Vol.* 1 (Eds. C. P. TSOKOS and I. N. SHIMI).

BASU, A. P. and EBRAHIMI, N. (1984). Testing whether survival function is bivariate new better than used. *Commun. Statist. – Theor. Meth.*, **13** (15), 1839–1849.

BUCHANAN, W. B. and SINGPURWALLA, N. D. (1977). Some stochastic characterization of multivariate survival. *Theory and Application of Reliability Vol.* 1 (Eds. C. P. TSOKOS and I. N. SHIMI), 329–348, Academic Press, New York.

HOEFFDING, W. A. (1948). A class of statistics with asymptotically normal distribution. *Ann. Math. Statist.*, **19**, 293–325.

HOLLANDER, M., PARK, D. H. and PROSCHAN, F. (1986). A class of life distributions for aging. *J. Amer. Statist. Assoc.*, **81**, 91–95.

MARSHALL, A. W. and OLKIN, I. (1967). A multivariate exponential distribution. *J. Amer. Statist. Assoc.*, **62**, 291–302.

RYZHIK, I. M. and GRADSHTEYN, I. S. (1957). *Tables of Series, Products and Integrals.* VEB Deutscher Verlag der Wissenschaften, Berlin.

SEN, P. K. (1977). Some invariance principles relating to Jackknifing and their role in sequential analysis. *Ann. Statist.*, **5**, 316–329.

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